Tolerance of Informality and Occupational Choices in a Large Informal Sector Economy

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April 1, 2021

Abstract

We study an equilibrium two-sector occupational choice model - agents can be either (formal or informal) entrepreneurs or workers. An informal entrepreneur is subjected to taxation determined by the combination of her capital choice and the society's tolerance of informality. Our model is consistent with many empirical findings regarding the informal sector in Brazil, a developing economy with a large informal sector. With a calibrated version of our model, we show that as the society's tolerance of informality decreases, the informal sector employs less capital and labor inputs and produces less output - informality decreases. We conduct several counterfactual exercises. Informality is substantially lower in economies that are less tolerant of informal activities, formal entrepreneurs have more access to financial markets and taxation of output and labor is lower. We also extend the model to consider a stochastic taxation of informal activities - uncertainty regarding informal output taxation reduces informality.

KEYWORDS: Informal Sector; Social Norms; Credit Constraints; Limited Enforcement. JEL CLASSIFICATION: E6; E26; O11; O17; H26; Z13.

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1 Introduction

In this paper, we study an equilibrium occupational choice model in which agents can choose to become an entrepreneur or a worker either in the formal or in the informal sector. Agents are heterogeneous in their wealth and in their ability to manage a firm. Formal and informal firms transform physical capital and labor inputs into a single good using capital- and labor-intensive technologies, respectively. Formal and informal entrepreneurs can use their own resources to finance capital used in production. Only formal sector firms have access to the financial markets - an exogenous large number of financial intermediaries. Workers are paid the same competitive wage rate in both sectors and the economy's interest rate is determined endogenously. Government collects taxes on the formal, the informal output and on labor to finance informal sector monitoring costs. The main novelty of this paper is to study how an endogenous taxation of informal output affects the production distribution in the economy (formal vs. informal) and the occupational choices of its agents (entrepreneurs vs. workers).

In our framework, an informal entrepreneur is subjected to a tax rate, which is determined by the combination of her own choice of the capital input and the society's tolerance of informality. The informal output tax in our model can be understood as a *catch-all* variable that accounts for the actual taxation of informal activities as well as various other factors at play in the economy related to the detection and punishment of such activities. There are two reasons for this approach. First, informal firms tend to operate with lower levels of capital input to reduce their visibility and, thus, the chances of being detected by the tax authority. The more capital is used in production, more visible informal entrepreneurs tend to be and the higher is the taxation of their output. Second, we acknowledge the fact that social norms may impose restrictions on the government's punishment of informal activities. For instance, the more tolerant of informality a society is - either in the form of production or consumption of goods produced in the informal sector - the lower the taxation (or punishment) of informal activities. Hence, our modelling approach and numerical exercises capture the effects of a society's tolerance of informal activities and informal entrepreneurs own perception of social norms and informal taxation on production levels and occupational choices. The former is captured by an exogenous parameter calibrated for a large informal sector economy (Brazil), while the latter is expressed in the informal entrepreneur's optimal decision taking the society's tolerance of informality as given.

The combination of these two features affect the informal entrepreneur's maximization problem and, hence, the general equilibrium effects of policy changes. We consider two cases. First, the taxation of informal production is deterministic (our benchmark) and all informal firms are subject to the same tolerance of informality. Then, we extend the model to consider a stochastic taxation of informal activities. That is, being caught by the tax authority managing an informal production technology is a stochastic event. All informal entrepreneurs are inspected and they are forced to pay a tax that depends on the size of the firm and on the (heterogeneous) tax auditors' tolerance of informal activities. In this environment, informal entrepreneurs face the same probability of being caught by either a more or a less tolerant tax auditor.

Our model is consistent with many empirical findings regarding the informal sector in Brazil. The size of the informal sector (% GDP) ranges between 32.6% - 41.7% in the period 1991-2015 (Medina and Schneider, 2018). Data from the 2003 *Brazilian Informal Urban Economy Survey* (*ECINF*) suggest that the informal sector is largely represented by very small firms with at most five employees. Moreover, formal firms employ 84% more workers, 385% more capital than the informal ones, and their productivity is higher (Ulyssea, 2018). Using microdata from the 2008 *Brazilian National Household Sample Survey - PNAD*, a repeated cross section representative at the national level, we find that the fraction of individuals in the labor force who employ at least one worker is about 2% and self-employment accounts for 10% of the labor force. According to this survey, the informal sector share in the total employment ranges between 32.5% - 43.6% (2002-2012). We follow Antunes et al. (2015b) and define entrepreneurs as those who manage a labor force with income higher than the minimum wage (R\$415; 2008). Hence, in Brazil the percent of entrepreneurs in the labor force is about 7.6%. With a calibrated version of our model, we explore the quantitative implications of policy changes for agents' occupational choices, input allocations and production in the formal and informal sectors.

We show that as the society's tolerance of informality decreases labor and output falls in the informal sector, i.e., informality decreases. Because the taxation of informal activities is endogenously determined by how much capital informal entrepreneurs use, a less tolerant society imposes a higher taxation per unit of (informal) capital used. And, interestingly, as the society becomes less tolerant to informality, the observed production increase in the formal sector occurs mostly through the intensive margin channel. On the other hand, the decrease in the informal sector production is due to less agents working fewer hours (both extensive and intensive margins). Regarding the distribution of occupational choices, we observed that changes in the tolerance parameter lead agents to move across occupations - informal entrepreneurs become workers rather than across the formal-informal sector dimension.

Changes in labor income and formal output taxation have interesting effects on agents' occupational choices. While a reduction in the labor income tax leads informal entrepreneurs to change their occupation to become workers - as this now represents a higher payoff - a change in the taxation of the formal output increases the profitability of formal sector entrepreneurs. A lower tax on formal output not only attracts informal entrepreneurs to switch and become formal entrepreneurs (of a less labor-intensive production technology) but also leads those already operating in the formal sector to expand their production by hiring more workers and employing more capital.

More accessibility to financial markets has two main effects. First, formal sector entrepreneurs have more access to credit in order to finance production. This leads to a drop in the informal sector production, which is more than compensated by an increase in the formal output. And, the overall effect of more access to additional funds leads to more production in the more efficient sector and, consequently, more output and consumption. Second, the equilibrium wage increases, making the worker occupation more attractive for some informal entrepreneurs. Through this channel the size of the informal sector falls in both the output and the employment dimensions.

The results presented and discussed so far relied on the assumption of a deterministic taxation of the informal output. We then extended our approach to consider a stochastic taxation of informal production. We conduct numerical exercises in which now informal entrepreneurs face uncertainty regarding the tax auditor's tolerance of informal activities, i.e., whether a more or less tolerant tax auditor will inspect their businesses. Overall, uncertainty regarding informal output taxation reduces informality. In particular, when the society (tax auditors) is less tolerant of informal activities the share of informal entrepreneurs and informal production are smaller relative to the deterministic (benchmark) case. The sharpest contrast between the deterministic and the stochastic case is in the equilibrium interest rate, which is higher when informal entrepreneurs face a higher level of uncertainty regarding the taxation of their output and they manage firms in a society that is less tolerant to their activities.

Related Literature. There is an extensive literature in economics that studies both theoretically and empirically the causes (e.g., low level of human capital, poverty, institutions, social norms, taxation, government regulations, lack of access to finance) and consequences (e.g. poor provision of public goods, income inequality, low tax revenue) of informality, particularly in poor and developing countries. A non-exhaustive list of papers that focus on informality and topics of interest is: contract enforcement (Quintin, 2008), productivity (D'Erasmo and Boedo, 2012), economic development (La Porta and Shleifer, 2014), unemployment benefits (Bosch and Esteban-Pretel, 2015), search frictions in the labor market (Ciccarone et al., 2016), growth (Maiti and Bhattacharyya, 2020), and tax collection Di Caro and Sacchi (2020). See Schneider and Enste (2000) for a review of the economic literature on informal activities and also Gerxhani (2003) and Ulyssea (2020).

This paper is more directly connected to four main strands of the literature. First, there are studies on individuals' occupational choice decision to become either an informal entrepreneur or an informal worker. In this literature, our article is closely related to Antunes and Cavalcanti (2007) and Amaral and Quintin (2006). Antunes and Cavalcanti (2007) solve numerically a general equilibrium model with credit constrained heterogeneous agents, occupational choices over formal and informal businesses, financial frictions and a government sector which imposes taxes and regulations on formal firms. They find that contract enforcement and regulation costs are equally important to account for the size of the informal sector in a developing country. Amaral and Quintin (2006) model the costs associated with informal sector production as resulting from financial frictions. Managers choose to enter the formal sector when the return to outside financing exceeds the additional tax cost they must bear. As a result, the most productive managers self-select into the formal sector, and operate with more capital. We contribute to this literature

by considering an endogenous taxation of informal entrepreneurs, based on their capital input decision and the society's tolerance of informality.

Studies have shown that taxation, broadly speaking, is one of the main drivers of informality (e.g., Cerda and Saravia, 2013; López, 2017). On one hand, higher (lower) taxes can discourage (encourage) formal activities and push agents - workers and entrepreneurs - toward (away from) the informal sector. For instance, Saracoğlu (2008) shows that by reducing formal labor income taxation a country can successfully reduce employment in the informal sector - a result also observed in our analysis. On the other hand, auditing procedures, penalty and fines applied to those caught operating in the informal sector can potentially deter tax evasion and underground activities. Tied to this discussion is the notion of tax morale - the intrinsic or moral obligation to pay taxes, which points to the link between the quality of public policies and social values as potentially a key mechanism behind the low tax compliance rate and high informality observed in many economies (see, for instance, Kolm and Larsen, 2002; Boeri and Garibaldi, 2005; Torgler and Schneider, 2009a; Traxler, 2010; Varvarigos, 2017).

As a matter of fact, informality and tax evasion can be approached from many perspectives: they can be view as a problem of public finance, law enforcement, labor supply or ethics, or a combination of all these (Andreoni et al., 1998). According to the traditional economic approach of tax compliance, e.g., Allingham and Sandmo (1972), taxes are paid or evaded strategically. The taxpayer determines how much tax to pay (evade) as if making a gambling decision in which the higher expected returns resulting from evasion are balanced against the risk associated with the possibility of being caught and penalized. However, many studies have noted that levels of informality and tax evasion are far different than a risk vs. return model would predict (Skinner and Slemrod, 1985; Slemrod, 1992; Torgler, 2007; Alm et al., 2010). Researchers have noted that taxpayers exhibit a diverse range of beliefs and behaviors regarding the payment or evasion of taxes. Individuals do not always behave as the selfish, rational, self-interested individuals portrayed in the standard neoclassical paradigm, but rather are often motivated by many other factors (Alm et al., 1992; Alm and Torgler, 2011).

In this paper we assume that factors associated with informal activities and on how a society views informality can be translated into a punishment of (tax on) informal activities. An individual's tax behavior can then be seen as the outcome of the interaction of objective, external factors (e.g., the tax system as an imposed system) and subjective, person-bound factors such as personality and taxpayers interdependence with others (Groenland and van Veldhoven, 1983). Individuals are rarely in isolation as all are members of social groups, societies and cultures. Consequently, tax behavior is not a function purely of individual choice: individuals might look to others in order to decide what is acceptable, reasonable, and expected within the social context in which the action is made (Cullis and Lewis, 1997; Pickhardt and Prinz, 2014). It is in this context that we model and study the society's tolerance of informality and its economic implications.

In particular, there is growing evidence that, among other factors, individuals are influenced

by the social context in which decisions are made. As Alm (2019) points out, much individual behavior can be broadly viewed as a "psychological contract" between individuals (and also between individuals and government). Central to this contract is the broad notion of a social norm - a pattern of behavior that is judged in a similar way by others and that is sustained in part by social approval or disapproval (Acemoglu and Jackson, 2017). While informality might be tolerated and, to some degree, accepted in some societies, in others, informal activities are perceived as immoral, even illegal.¹

Third, several papers study the relationship between informality and financial development (e.g., Antunes and Cavalcanti, 2007). Antunes et al. (2008b) shows that differences across countries in intermediation costs and enforcement generate differences in occupational choice, firm size, credit, output and income inequality. Blackburn et al. (2012) study the relationship between the informal sector and financial development in a model of tax evasion and bank intermediation. The key implication of their analysis is that the marginal net benefit of income disclosure increases with the level of financial development. Guo and Hung (2020) find a positive correlation between financial development and the ratio of tax revenue over GDP; a result similar to ours.

Finally, there are several papers that study the Brazilian economy and features of its informal sector. In Ulyssea (2018) informal firms coexist with formal firms which may hire informal workers. Through counterfactual exercises, the author shows that there is substantial heterogeneity in policy effects between groups (switchers, always formal, and always informal firms) and within groups. De Paula and Scheinkman (2011) test implications of a simple equilibrium model of informality using data from the ECINF and verify that formal activities are positively correlated with firms' size and informal firms employ a lower capital-labor ratio. Using a nationally representative Brazilian panel data that covers both formal and informal workers, Gomes et al. (2020) study labor earnings dynamics and document that informality in Brazil is associated with more volatile earnings, while formal sector workers are subject to significant downside risk. See also Monteiro and Assuncao (2012), de Paula and Scheinkman (2010) and Engbom et al. (2021).

Besides this introduction, this paper is organized in three additional sections. Section 2 presents the model. In Section 3 we present the results for a calibrated version of the model and conduct counterfactual analyses. Section 4 concludes.

2 The Economy

The economy is populated by one-period lived agents in discrete time. Every period, a cohort of measure one is born and the economy goes on forever. Agents are heterogeneous with respect to

¹A growing literature has considered other potential explanations for individuals' tax compliance behavior (Pickhardt and Prinz, 2014). Factors that might affect an individual's decision to pay or evade taxes and, hence, engage in informal activities, include ethics (e.g., Alm and Torgler (2011)), institutional quality (e.g., Torgler and Schneider (2009b); Alm et al. (2012)) and social interactions (e.g., Myles and Naylor (1996); Kirchler (2007); Fortin et al. (2007); Coricelli et al. (2010); Dulleck et al. (2016)).

their endowments and their ability to manage a firm. Agent's wealth is inherited from her previous generation but her entrepreneurial ability is not. Agents are endowed with one unit of time and they can choose to become either an entrepreneur or a workers.

If an agent decides to become an entrepreneur she produces a single final good managing either a formal or an informal production technology - i.e., formal and informal firms combine labor and capital to produce the same good with different technologies. Firms in both sectors face different taxation and credit constraints. Only formal sector firms have access to the financial markets, which is represented by an exogenous large number of financial intermediaries. These intermediaries rent agents' wealth and lend it at an endogenously determined interest rate. The final good can be either consumed, invested and left as bequests for the next generation. Its price is normalized to one. The decision to become an entrepreneur and the firm's size depend on an agent's ability to manage a firm, her inherited wealth, her access financial markets and output taxation. Instead of managing a firm, agents can work in either formal or informal firms, which pay the same competitive wage rate. A worker does not value leisure and, hence, he inelastically supplies labor.

The government taxes workers and formal firms. Due its limited and costly monitoring capacity, the government taxes informal firms only partially. Entrepreneurs caught operating in the informal sector are subject to a tax rate that depends on the size of its (informal) capital stock, as well as on the society's tolerance of informal activities. The government tax revenue finance transfers to agents and monitoring costs. We assume, without loss of generality, that the society's tolerance of informality is common knowledge and capital fully depreciates.

2.1 Preferences and Technologies

The timing of the model is as follows. At the beginning of period t, the agents inherit wealth b_t from their parents, which follows from an endogenously determined wealth distribution G_t .² They also draw an entrepreneurial ability x from a distribution F(x). Next, agents choose their occupations (entrepreneur or worker) and the sector of activity (formal or informal). Production takes place. Based on the occupational choice payoffs, all agents then make optimal consumption and wealth decisions. The government taxes workers and formal entrepreneurs. It also monitors and taxes informal sector entrepreneurs. At the end of that period t agents die and they are replaced by their heirs. And, these steps are repeated from t + 1 on.

In our economy, agents value their current consumption c_t and the amount of wealth b_{t+1} they leave for their offspring (Banerjee and Newman, 1993). The agent's preferences are represented by

²In the first period of the economy (t = 0), wealth b_0 is drawn from a initial (exogenous) wealth distribution G_0 . In Section 2.5 we discuss how wealth evolves over time. We follow a large literature that assumes that wealth and ability distributions are independent (Benhabib et al. (2011)). We acknowledge that the correlation of wealth and ability could play an important role in our analysis, but this is beyond the scope of this paper and we leave it for future research.

the following utility function

$$u(c_t, b_{t+1}) = c_t^{\eta} b_{t+1}^{1-\eta},\tag{1}$$

where $\eta \in (0, 1)$ represents the weight of current consumption on the agent's instantaneous utility.

If an agent decides to become an entrepreneur, she combines labor (l) and capital (k), along with her entrepreneurial ability x to produce the same good either in the formal (i = F) or in the informal (i = I) sector, according to the following production technology

$$y_i = x A_i k_i^{\alpha_i} l_i^{\beta_i}, \tag{2}$$

where $\alpha_i, \beta_i \in (0, 1)$ and k_i, l_i and A_i are the capital and labor inputs and the productivity in sector i = F, I, respectively. We normalize $A_F = 1$ and, in line with the literature, we assume that productivity is lower in the informal sector $(A_I < 1)$. Production technologies exhibit decreasing returns to scale, i.e., $\alpha_i + \beta_i < 1.^3$ We further assume that $\alpha_I < \alpha_F$, which implies that production in the informal firms is more labor intensive.

2.2 Workers' and Entrepreneurs' Problems

We now consider the problem faced by workers and entrepreneurs. In a given period t, an agent has to decide whether to become a worker or an entrepreneur. An agent with inherited wealth b_t that decides to become a worker inelastically supplies labor to a firm in either the formal or in the informal sector, taking the wage rate w_t as given. Hence, the worker's payoff $\Pi_w(b_t; w_t, r_t)$ is given by

$$\Pi_w(b_t; w_t, r_t) = (1 - \tau_w)w_t + (1 + r_t)b_t + T_t,$$
(3)

where τ_w is the tax rate on labor income, r_t is the rate of return on households' savings, w_t is the wage rate and T_t is a lump-sum government transfer. The wage rate and the lump-sum transfers are the same regardless whether the agent work in the formal or in the informal sector.

Instead of supplying labor services, an agent can choose to become an entrepreneur and manage either a formal or an informal firm. One interpretation for this choice is the decision by entrepreneurs whether or not to legally declare their establishment (Amaral and Quintin, 2006). An entrepreneur's goal is to maximize profit by producing and selling the final good according to the sector-specific production function, equation (2), subject to labor and capital costs and output taxation. Thus, the profit maximization problem of an entrepreneur (e) managing a firm i = F, Iis as follows:

$$\pi_{e,i}(b_t, x_t; w_t, r_t) = \max_{k_i, l_i \ge 0} \left\{ (1 - \tau_i) y_i - w_t l_i - (1 + r_t) k_i : 0 \le k_i \le \lambda_i b \right\}.$$
(4)

³Basu and Fernald (1997) find that a typical industry appears to have significantly decreasing returns to scale.

And, the entrepreneur's payoff Π_e is given by

$$\Pi_e(b_t, x_t; w_t, r_t) = \pi_{e,i}(b_t, x_t; w_t, r_t) + (1 + r_t)b_t + T_t,$$
(5)

which also takes into account the return on the entrepreneur's own financial resources $(1 + r_t)b_t$ and government transfers T_t . Two features of a firm's profit maximization problem, equation (4), deserve particular attention, namely, the credit (collateral) constraint, i.e. $0 \le k_i \le \lambda_i b$, and output taxation τ_i .

In our model, credit markets are assumed to be imperfect and all borrowing and lending decisions are made through financial intermediaries. The amount of capital k_i used in production combine the entrepreneur's own capital and capital borrowed from financial intermediaries. A capital constrained formal entrepreneur can obtain additional funds but due to the imperfect enforceability of contracts (Evans and Jovanovic, 1989), the access to additional units of capital is determined by entrepreneurs' own wealth through a collateral constraint $0 \le k \le \lambda_F b$, where $\lambda_F \ge 1$. The parameter λ_F informs the entrepreneur's accessibility to the financial markets, which can be interpreted, for instance, as the economy's degree of financial development (Buera et al., 2015). If $\lambda_F = \infty$, the credit market is perfect and there is no barriers to indebtedness. On the other hand, when $\lambda_F = 1$ the firm's capital is financed by the entrepreneur's own resources. This latter condition represents the case of informal entrepreneurs in our economy. In line with most of the literature that studies financial frictions and informal activities, we assume that informal entrepreneurs do not have access to the financial markets ($\lambda_I = 1$).

We assume that the government levies taxes τ_i on the firm's output (Hsieh and Klenow, 2009; Restuccia and Rogerson, 2008). Formal entrepreneurs are subject to an exogenously given tax rate τ_F on the their output. Tax collection in the formal sector is straightforward as production can be directly and costlessly observed by the tax authority. On the other hand, the government can only monitor informal production imperfectly. Taxation of informal output is endogenously determined by the amount of capital k_I used in production along with a parameter that reflects the government's ability to tax informal entrepreneurs. Hence, an informal entrepreneur is subject to the following tax rate:⁴

$$\tau_I = 1 - e^{-k_I \zeta} \tag{6}$$

where the parameter $\zeta \geq 0$ is assumed to be a proxy for the fact that social norms impose restrictions on the government's ability to tax informal activities (see Sandmo (2005) and citing literature). In other words, ζ captures the combination of a society's intolerance of informality as well as the informal entrepreneur's perception of how informal activities are tolerated by society and

⁴The government could potentially discourage informal activities if it had access to either higher detection probabilities or very harsh penalties. However, detection probabilities are typically low because of social norms that limit "cruel and unusual punishments". We take into account such limitations in our parameter ζ . Also, note that we could easily adapt our benchmark model to consider the monitoring intensity as an stochastic variable instead of a deterministic one, as assumed here. We consider this possibility in Section 3.3.

how informal entrepreneurs are taxed (punished). For instance, in one extreme case of an economy where informal activities are fully accepted (tolerated), $\zeta = 0$ implies that $\tau_I = 0$ and the informal sector production is not taxed. The more tolerant a society is, the lower ζ , which renders a lower τ_I and higher net informal profits. On the other hand, a higher value of ζ represents a society that is less tolerant to informality and, hence, imposes a harsher punishment on informal entrepreneurs.⁵ Hence, the role of τ_I in our model is to capture the joint effect of society's tolerance of informal production (social norms) and how informal entrepreneurs themselves perceive the punishment imposed by the government. These are reflected on the informal entrepreneurs (endogenous) choice of the capital input. The combination of these two features affect the informal entrepreneur's maximization problem and, hence, the general equilibrium effects of policy changes.

A formal entrepreneur's profit maximization problem, equation (4), imply the following optimal capital demand functions:

$$k_F = \begin{cases} \left[x(1-\tau_F) \left(\frac{\alpha_F}{1+r}\right)^{1-\beta_F} \left(\frac{\beta_F}{w}\right)^{\beta_F} \right]^{\frac{1}{1-\alpha_F-\beta_F}}, & \text{if } k_F \le b, \\ \lambda_F b, & \text{otherwise,} \end{cases}$$
(7)

otherwise,
$$(8)$$

and the optimal labor demand functions:

$$d_F = \left\{ \left[x(1-\tau_F) \left(\frac{\alpha_F}{1+r}\right)^{\alpha_F} \left(\frac{\beta_F}{w}\right)^{1-\alpha_F} \right]^{\frac{1}{1-\alpha_F-\beta_F}}, \text{ if } k_F \le b, \right.$$
(9)

$$\left[\left[x(1-\tau_F)(\lambda_F b)^{\alpha_F} \left(\frac{\beta_F}{w}\right) \right]^{\frac{1}{1-\beta_F}}, \quad \text{otherwise.} \quad (10)$$

Notice that equations (7)-(10) highlight the fact that we have two types of formal entrepreneurs - those constrained by their own resources but with access to the financial markets, and those unconstrained. Taxation of formal output (τ_F) and the formal entrepreneur's accessibility to the financial markets, measured by the collateral constraint parameter ($\lambda_F \geq 1$), affect the firm's optimal capital and labor allocations, and, thus, the optimal formal firm's profit. The optimal capital and labor demand decisions of unconstrained entrepreneurs are represented in equations (7) and (9), respectively. In other words, the entrepreneurs with optimal capital demand $k_F \leq b$ constitute the mass of self-financed formal entrepreneurs. On the other hand, if the optimal capital demand of an entrepreneur is greater that her own resources b (i.e., $b < k_F \leq \lambda_F b$) she will finance

⁵In our model, the informal output tax, τ_I can be understood as a *catch-all* variable that accounts for the actual taxation of informal activities as well as various other factors at play in the economy related to the detection and punishment of such activities. For instance, we can also interpret the "punishment" of informal activities not just necessarily tied to taxation of informal output. There is large evidence that informal activities are associated or subject to corruption, weak rule of law and business institutions in general. By operating low scale firms, informal entrepreneurs might avoid engaging in side deals with tax inspectors and cumbersome bureaucracy, but even so be subject to overall costs - e.g., transportation - that lead to production losses.

production with additional resources through the financial markets.⁶ The optimal capital and labor demand decisions of constrained entrepreneurs that have access to the financial markets are represented by equations (8) and (10), respectively.

Informal entrepreneurs are constrained by their own resources when making their optimal capital and labor choices $(k_I \leq b; \lambda_I = 1)$. Taking into account the taxation of the informal output, equation (6), they maximize profits, equation (4), which implies the following optimal demand functions for informal capital and labor, respectively:

$$k_{I} = \begin{cases} \left[xA_{I} \left(\frac{(1-\tau_{I})\beta_{I}}{w} \right)^{\beta_{I}} \left(\frac{(1-\tau_{I})\alpha_{I}-\tau_{I}\epsilon}{1+r} \right)^{1-\beta_{I}} \right]^{\frac{1}{1-\alpha_{I}-\beta_{I}}}, & \text{if } k_{I} \le b, \\ b, & \text{otherwise,} \end{cases}$$
(11)

otherwise,
$$(12)$$

$$l_I = \left\{ \left[x A_I \left(\frac{(1-\tau_I)\beta_I}{w} \right)^{1-\alpha_I} \left(\frac{(1-\tau_I)\alpha_I - \tau_I \epsilon}{1+r} \right)^{\alpha_I} \right]^{\frac{1}{1-\alpha_I - \beta_I}}, \quad \text{if } k_I \le b,$$
(13)

$$\left(\left[x A_I (1 - \tau_I) b^{\alpha_I} \left(\frac{\beta_I}{w} \right) \right]^{\frac{1}{1 - \beta_I}}, \quad \text{otherwise.} \quad (14)$$

where $\epsilon = (\partial \tau_I / \partial k_I) (k_I / \tau_I)$ is the elasticity of the informal tax with respect to the informal capital. Unconstrained (constrained) informal entrepreneurs optimal capital and labor demand decisions are represented by equations (11) and (13) (equations (12) and (14)), respectively. Notice that, facing a higher taxation of the informal output, unconstrained informal firms reduce the optimal amount of capital input in production, equation (11), consequently lowering their optimal labor demand, equation (13).

2.3Agent's Optimal Occupational Choice

In the previous section, we presented the optimal payoffs of workers and (formal, informal) entrepreneurs, equations (3) and (5), respectively. Taking prices, income taxation and formal and informal output taxation as given, an agent with an entrepreneurial ability x and wealth b_t decides her occupational choice. That is, she must decide whether to become a worker or an entrepreneur and, in the latter case, whether to manage a formal or an informal production technology.

The agent's optimal occupational choice is the one that generates the highest payoff, i.e., the solution of the following maximization problem

$$\Pi(b, x; w, r) = \max \left\{ \Pi_w(b; w), \Pi_e(b, x; w, r) \right\},$$
(15)

where $\Pi_w(b; w)$ and $\Pi_e(b, x; w, r)$ are given by equations (3) and (5), respectively.

Given the distributions of entrepreneurial ability and wealth, the solution of the agent's prob-

⁶It is straightforward to show that entrepreneurs that obtain additional funds from financial intermediaries invest all their capital endowments in their firms, see Antunes et al. (2008a).

lem, equation (15), allow us to characterize the mass of entrepreneurs $\mathbb{E}(w, r)$ and workers $\mathbb{W}(w, r)$ in the economy, respectively,

$$\mathbb{E}(w,r) = \{(b,x) \in \mathcal{O} : \max\{\Pi_F(b,x;w,r), \Pi_I(b,x;w,r)\} \ge \Pi_w(b;w)\},$$
(16)

$$\mathbb{W}(w,r) = \{(b,x) \in \mathcal{O} : \Pi_w(b;w) > \max\{\Pi_F(b,x;w,r), \Pi_I(b,x;w,r)\}\},$$
(17)

where $\mathcal{O} = [0, \infty) \times [x_L, x_H]$ and $\mathbb{E}(w, r) + \mathbb{W}(w, r) = 1$. And, the mass of formal and informal entrepreneurs are defined as follows, respectively

$$\mathbb{E}_F(w,r) = \{(b,x) \in \mathbb{O} : \{\Pi_F(b,x;w,r) \ge \Pi_I(b,x;w,r)\}\} \cap \mathbb{E}(w,r), \tag{18}$$

$$\mathbb{E}_{I}(w,r) = \{(b,x) \in \mathcal{O} : \{\Pi_{I}(b,x;w,r) > \Pi_{F}(b,x;w,r)\}\} \cap \mathbb{E}(w,r),$$
(19)

where $\mathbb{E}(w, r)$ is defined in equation (16) and $\mathbb{E}_F(w, r) \cup \mathbb{E}_F(w, r) = \mathbb{E}(w, r)$.

2.4 Agent's Utility Maximization Problem

Given the agent's optimal occupational choice, she chooses current consumption c_t and the amount of wealth b_{t+1} she will leave for her offspring. Recall that, although entrepreneurial ability is drawn every period from the same distribution, the wealth distribution evolves over time. Thus, there is a link between generations that occurs through an agent's optimal wealth decision - parents decide to accumulate and transfer wealth to their children, which may affect their occupational choices.

Hence, the agent's optimization problem is to maximize utility, equation (1), subject to the following budget constraint:

$$c_t + b_{t+1} \le \Pi(b, x; w, r),$$
(20)

where $\Pi(b, x; w, r)$ is given by equation (15). The solution of the agent's utility maximization problem implies that the optimal current consumption c_t and next period wealth b_{t+1} are, respectively, $c_t = \eta \Pi(\cdot)$ and $b_{t+1} = (1 - \eta) \Pi(\cdot)$.

2.5 Wealth Distribution

It is important at this point to describe how wealth evolves over time. In order to characterize the law of motion of the wealth distribution, we assume that G_0 and G_t are the initial and time tdistributions of wealth, respectively. Let $b \in Z = [b_L, b_h] \subset \Re_+$ represent the time t individual's wealth inherited from a previous generation (say, her parents). We assume that \mathcal{Z} is a σ -algebra in Z and G is a probability measure defined on the measurable space (Z, \mathcal{Z}) . Note that G characterizes the cross-sectional distribution of wealth among individuals. That is, for any $V \subset Z$, with $V \in \mathcal{Z}$, G(V) describes the mass of individuals with wealth defined in Z. Thus, for any $(b, V) \in (Z, \mathcal{Z})$, a non-stationary transition probability function P_t is defined as follows:

$$P_t(b, V) = Pr[b_{t+1} \in V|b_t].$$
(21)

In other words, for $V \in \mathbb{Z}$ and $b \in \mathbb{Z}$, the function $P_t(b, V)$ defines the probability that an individual's wealth will be in the set V in the period t + 1, given that her wealth (state) in period t is b. Then, the law of motion of the wealth distribution is given by:

$$G_{t+1} = \int P_t(b, V) G_t(db).$$
(22)

2.6 Government and the Economy's Resource Constraint

The government finances transfers T to entrepreheurs and workers and a *per* informal firm monitoring cost M through formal and informal output taxation $(\mathbb{T}_i, i = F, I)$ and labor income tax on workers (\mathbb{T}_w) . The government budget constraint is as follows

$$\mathbb{T}_F + \mathbb{T}_I + \mathbb{T}_w = \iint_{\mathbb{X}_{EW}} TF(dx)G_t(db) + \iint_{\mathbb{X}_{E_I}} MF(dx)G_t(db)$$
(23)

where $\mathbb{X}_{EW} = (x, b) \in \mathbb{E}(w_t, r_t) \cup \mathbb{W}(w_t, r_t), \ \mathbb{X}_{E_I} = (x, b) \in \mathbb{E}_I(w_t, r_t),$

$$\mathbb{T}_i = \iint_{\mathbb{X}_{E_i}} \tau_i y_i F(dx) G_t(db), \quad \mathbb{T}_w = \iint_{\mathbb{X}_W} \tau_w w F(dx) G_t(db),$$

 $\mathbb{X}_{E_i} = (x, b) \in \mathbb{E}_i(w_t, r_t)$, for i = F, I, and $\mathbb{X}_W = (x, b) \in \mathbb{W}(w_t, r_t)$.

The economy's resource constraint is

$$\iint_{\mathbb{X}_{E}} yF(dx)G_{t}(db) = \iint_{\mathbb{X}_{EW}} cF(dx)G_{t}(db) + \iint_{\mathbb{X}_{EW}} hF(dx)G_{t}(db) + \iint_{\mathbb{X}_{EW}} TF(dx)G_{t}(db) + \iint_{\mathbb{X}_{E_{I}}} MF(dx)G_{t}(db)$$
(24)

where, abusing notation, $y = y_F + y_I$ and $h \equiv b_{t+1}$. The total amount of resources in this economy (left-hand side of equation (24)) is equal to the sum of current consumption, next period wealth, government transfers to entrepreneurs and workers and informal sector monitoring cost (right-hand side of equation (24)).

2.7 The Stationary Equilibrium

We are now ready to present our definition of a stationary equilibrium for our economy.

Definition 1. A stationary competitive equilibrium is a policy set $\Upsilon = \{\tau_F, \tau_I, \tau_w, T, M\}$ that includes a tax on the formal output, a tax on the informal output which is a function of the society's tolerance of informal production (ζ), a tax on the worker's income, transfers to workers and entrepreneurs and per firm monitoring costs, respectively; a price system $Q = \{w_t, r_t\}$ of wages and interest rate; agents' allocations $X = \{c_t, b_{t+1}\}$, i.e., current consumption and next period wealth, the degree of financial markets accessibility (λ) , a distribution of entrepreneurial ability F(x), an initial wealth distribution G_0 and an invariant wealth distribution G(b) such that, at the steady-state

- 1. The resulting optimal allocations satisfy the agents' optimal occupational choice described in equations (3), (4), (5), (6), and (15),
- 2. The optimal allocations maximizes the individuals' utility, equation (1), subject to a budget constraint, equation (20),
- 3. The wealth distribution law of motion is given by equation (22), combined with equation (21),
- 4. The government budget constraint and the economy's resource constraint, equations (23) and (24), respectively, are satisfied, and
- 5. The wage rate and the economy's interest rate satisfy the following market clearing conditions, respectively:

$$\iint_{\mathbb{X}_W} F(dx)G(db) = \iint_{\mathbb{X}_E} lF(dx)G(db).$$
(25)

$$\iint_{\mathbb{X}_E} kF(dx)G(db) = \iint_{\mathbb{X}_{EW}} bF(dx)G(db).$$
(26)

where $l = l_F + l_I$ and $k = k_F + k_I$, $X_{EW} = (x, b) \in \mathbb{E}(w_t, r_t) \cup \mathbb{W}(w_t, r_t)$, $X_E = (x, b) \in \mathbb{E}_i(w_t, r_t)$, and $X_W = (x, b) \in \mathbb{W}(w_t, r_t)$

It can be shown that the steady-state equilibrium is unique and the economy converges to this equilibrium from any initial condition. See Antunes et al. (2008a) for details on the characterization of equilibrium.

3 Economic Implications of the Model

In this section we describe the quantitative implications of a calibrated version of our model. We calibrate the model to match important characteristics of the formal and the informal sectors, as well as aggregate features of the Brazilian economy. Then, we simulate the benchmark steady state equilibrium and conduct several counterfactual exercises. In particular, we study how changes in the society's tolerance of informality, taxation and access to the financial markets affect the individuals occupational choices and the aggregate behavior of the Brazilian economy.⁷

 $^{^{7}}$ We assume zero lump sum transfers in our numerical exercises. The government budget constraint, equation (23), is adjusted accordingly.

3.1 Calibration and Parameterization

To carry out our numerical exercises, first we calibrate seven parameters so that the stationary equilibrium is consistent with target moments describing the empirical distributions of informal output and employment, the economy total credit (%GDP), aggregate consumption (%GDP), total tax collection (%GDP), and the share of formal entrepreneurs in the labor force as well as other relevant data moments. These seven parameters are the informal sector labor share (β_I), the society's tolerance of informal activities parameter (ζ), the weight of consumption in the utility function (η), the parameters associated to the accessibility to the financial markets parameter (λ_F) and the informal sector productivity (A_I), the entrepreneurial ability and initial wealth distributions parameters (χ and ξ , respectively). We normalize the formal sector productivity parameter $A_F = 1$. We also choose values for the labor income tax (τ_W), the formal output tax (τ_F), formal sector capital (α_F) and labor (β_F) income shares based on information that is exogenous to the model and consistent with empirical studies in the literature, in particular, those related to the Brazilian economy.⁸ The calibrated values of the model parameters are summarized in Table I and each of these parameters is discussed in turn below.

Following Antunes and Cavalcanti (2007) and in line with Gollin (2002), we set α_F and β_F such that about 55% of formal income is paid to labor, 35% is paid to the remuneration of capital, and 10% are profits. Hence, as our benchmark, we set the capital and labor shares in the formal sector to $\alpha_F = 0.35$ and $\beta_F = 0.55$, respectively. Recall that production in the informal sector is also assumed to exhibit decreasing returns to scale, i.e., $\alpha_I + \beta_I < 1$. Consistent with the assumption that $\alpha_F > \alpha_I$ and that ten percent of informal income are profits, we fix $\alpha_I = 0.30$ as our benchmark value.⁹ According to the Brazil National Household Sample Survey (*PNAD*), informal workers represent 32.5 - 43.6% of the employed labor force in the period 2002-2012. We set the informal sector labor share in our model $\beta_I = 0.60$ to match the share of informal workers in the total employment in Brazil in the year 2008, which according to the *PNAD* is estimated to be 38.1%. Notice that an informal labor share greater than the one observed in the formal sector is consistent with, for instance, Loayza (1996) - in developing economies, informal firms tend to be labor intensive since capital is scarcer than labor.

Moreover, the inefficiency of informal sector production is well established in the literature (see, for instance, La Porta and Shleifer, 2014). Ulyssea (2018) shows that the informal sector productivity in Brazil is approximately 20% lower than its formal counterpart. Hence, given the normalized formal sector productivity parameter ($A_F = 1$) we set the informal sector parameter to $A_I = 0.8$ in our benchmark calibration. We set the utility function curvature parameter $\eta = 0.88$

⁸Using microdata from the 2008 Brazilian households survey (PNAD), Antunes et al. (2015a) find that the percent of people in the labor force who employ at least one worker is about 2%. Self-employment accounts for 10% of the labor force. However, it is hard to distinguish those self-employed who are managing a business or who are employed as a worker to avoid Brazil's strict labor laws and regulations.

⁹Our main results are robust to reasonable variations around this benchmark calibration. Results available upon request.

Parameter	Description	Source	Value		
Preferences					
η	Utility function curvature	(5)	0.880		
Technology					
$lpha_F$	Formal sector - Capital share	(1)	0.350		
β_F	Formal sector - Labor share	(1)	0.550		
$lpha_I$	Informal sector - Capital share	(6)	0.300		
β_I	Informal sector - Labor share	(5)	0.600		
A_F	Formal sector - Productivity	(6)	1.000		
A_I	Informal sector - Productivity	(2)	0.800		
Financial Ma	rket				
λ_F	Financial markets accessibility	(5)	10.000		
Tax Policies					
$ au_F$	Tax rate on output	(3)	0.340		
$ au_w$	Tax rate on labor income	(4)	0.275		
ζ	Tolerance of informal sector	(5)	1.000		
Talent and In	nitial Wealth Distributions				
χ	Talent (upper bound)	(5)	2.000		
ξ	Wealth (upper bound)	(5)	0.100		
Sources: 1. Gollin (2002); 2. Ulyssea (2018); 3. Fernández-Rodríguez					

Table I: Benchmark parameter calibration.

and Martínez-Arias (2014); 4. Ministry of Economy of Brazil; 5. Jointly calibrated; 6. Normalized.

so that the steady state equilibrium consumption to GDP ratio in our model is consistent with data from the Penn World Table (PWT) - in the period 1960-2017, aggregate private consumption corresponds to 73% of the Brazilian GDP.¹⁰

Brazil has a very complex production and labor income tax code, which is beyond the scope of this paper. We follow Fernández-Rodríguez and Martínez-Arias (2014) and set the formal output tax rate $\tau_F = 0.34$, i.e., formal output is taxed at a 34% rate. In the tax code, labor income is taxed at rates that range from zero to 27.5% (Ministry of Economy of Brazil). In our benchmark calibration, we set $\tau_w = 0.275$.¹¹ In the period 1991-2015, the size of the informal sector in Brazil is estimated to range from 32.6 - 41.7 percent of the Brazilian GDP according to Medina and Schneider (2018). We choose the society's tolerance of informal activities $\zeta = 1.0$ so that we match the estimated size of the informal sector to the GDP in the year 2008 (35%) in the stationary

¹⁰We use the real consumption to real GDP ratio, 2011, at constant national prices (2011 US\$ millions).

¹¹These values are also consistent with the estimated tax burden in the Brazilian economy. See Prado (2011) and Pereira and Ellery Júnior (2011) for more on this.

equilibrium of our model economy. The parameter that represents the entrepreneur's accessibility to the financial markets (λ_F) is chosen so that our model matches the observed credit to GDP ratio of 0.372 in Brazil (Central Bank of Brazil). This results in a value of $\lambda_F = 10$ in our benchmark calibration.

Entrepreneurs' ability (x) and agents' initial wealth (b) are not directly observed. In our benchmark calibration, we assume that both x and b are uniformly distributed with mean zero and independent of each other, i.e., $x \sim U(0, \chi)$ and $b \sim U(0, \xi)$; (see, for instance, Stiglitz, 1969; Benhabib et al., 2011). In order to calibrate the variance of the distributions (χ, ξ) , first notice that, as discussed in Section 2.5, we assume an exogenous wealth distribution only in the first period of our economy. As agents take this as given and make their consumption and wealth decisions, the wealth distribution becomes endogenously determined. Second, since the parameter ξ represents the agent with the highest initial wealth in our benchmark economy we set its value exogenously and derive a compatible entrepreneurial ability χ . We then check whether this (x, b)pair is compatible with the steady state profits an agent would receive if she were to become either a formal or an informal entrepreneur. For different values of ξ , we repeat this procedure several times and construct different ability-wealth pairs such that the optimal steady-state equilibrium is satisfied. Then, we allow each individual to solve her maximization problem and to choose the highest payoff occupation, i.e., the occupation that generates the best outcome to conclude this stage. Finally, we check whether the calibrated mass of entrepreneurs meets a selected target for the Brazilian economy, i.e., the share of formal entrepreneurs in the labor force. Hence, in our benchmark calibration, the upper bounds of the ability and wealth distributions are set to $\chi = 2.00$ and $\xi = 0.1$, respectively.

Table II presents our key target statistics for the Brazilian economy as well as those resulting from our calibrated model in a stationary equilibrium. Notice that our model matches the Brazilian economy fairly well along several dimensions. In particular, the model fits well the statistics related to the informal sector: output, employment and formal and informal (average) capital and labor input ratios.¹² In addition, our calibrated model is also consistent with two additional features observed in the data: the participation of formal entrepreneurs in the labor force (*PNAD*) and the standard deviation of the informal employment (*ECINF*).

Figure 1 illustrates the payoff and profit distributions according to our calibrated model and data from the Brazilian Informal Urban Economy Survey (*ECINF*), respectively. An important stylized fact of large informal sector economies is displayed in this figure, i.e., formal and informal sector firms might have the same payoff/profit. In Figure 1 this is highlighted by the overlap of formal and informal entrepreneurs' payoffs (model) and profits (*ECINF* data). The fact that informal entrepreneurship is an occupation that can generate payoffs similar to the one observed by formal entrepreneurs can be attributed to three main factors: a society's high tolerance of informal

¹²To capture the latter, we use formal and informal firms' data from the ECINF. Capital inputs were proxied by the variable total value of facilities and equipment.

	Brazilian	Benchmark
	economy	model
Occupational Choice (%)		
Workers	85.9	76.7
Formal entrepreneurs	7.5	8.9
Informal entrepreneurs	6.6	14.4
Informal sector		
Share total output $(\%)$	35.1	36.2
Share total employment $(\%)$	38.1	37.5
Standard deviation (labor)	0.73	0.51
Share of Total Output (%)		
Total tax collection	35.9	37.0
Total consumption	73.0	73.3
Total credit	37.1	34.0
Formal and informal ratios		
Averages of employment	1.84	1.67
Averages of capital	4.85	5.49

Table II: Key statistics: Data and benchmark Economy

Sources: Ministry of Economy of Brazil, Brazilian Central Bank, ECINF, Brazilian households survey (PNAD), Penn World Table 9.1, Medina and Schneider (2018) and Ulyssea (2018).

activities, a high (formal) tax burden and a labor intensive (informal) production technology. In addition, we observe that both distributions have a long right tail (fewer firms with large payoffs and profits), there is a large concentration of informal firms at low levels of capital, and the mass of informal workers is smaller than its formal counterpart.

The equilibrium entrepreneurial ability distribution, the wealth distribution and the payoff distribution are plotted in Figure 2. We divide the state space (b, x) into the set of workers, formal and informal entrepreneurs, W(w, r), $\mathbb{E}_F(w, r)$ and $\mathbb{E}_I(w, r)$, respectively. Notice that if an agent's entrepreneurial ability is low, her optimal occupational choice is to become a worker (the dark gray shaded area). Workers get paid the equilibrium wage regardless the sector they are employed at and we observe workers across the whole wealth distribution. Entrepreneurs are more concentrated at high levels of entrepreneurial ability, even for low levels of wealth. Accessibility to the financial markets also play a role in determining whether a formal entrepreneur has access to additional funds to finance her production. While informal entrepreneurs do not have access to the financial markets, by operating in the informal sector they avoid formalization costs, which in our model is represented by the formal output taxation.



Figure 1: Payoffs Distribution (baseline model) \times Profits Distribution (ECINF)

Notes: Left panel - our simulations are based on a random draw of 20,000 individuals from the ability and wealth distributions, according to our model. Right panel - ECINF informal firms' profits. The data is normalized (log) to allow for comparisons. The sum of the bars height is less than or equal to 1. That is, the figures display the relative frequencies

3.2 Implications of the Model and Quantitative Exercises

In this section, we conduct several quantitative exercises to evaluate the impact on economic outcomes of key parameter changes. In particular, we focus on how the formal and informal sector production and employment, tax collection, wage and interest rate are affected by a variety of parameter and policy changes, i.e., the society's tolerance of informality (ζ), the taxation of labor and formal output, τ_w and τ_F , respectively, and the entrepreneur's accessibility to the financial markets (λ_F). These results are presented in Tables III - VII. To allow for comparisons, columns marked with an asterik (*) in each table show the results for our benchmark parameterization. We then vary one parameter at a time while keeping all other parameters constant at their benchmark levels.

Tolerance of Informality. As the society's tolerance of informality decreases, i.e., ζ increases, the informal sector employs less labor input, produces less output, and informality decreases (Table III). The case of $\zeta = 1.5$ is illustrative - a tolerance level fifty percent lower than our benchmark value. In words, it means that a society is less tolerant with informality and, hence, a harsher punishment is imposed on informal entrepreneurs. Moreover, because the taxation of informal activities τ_I is endogenously determined by how much capital is used by informal entrepreneurs, a larger ζ imposes a higher taxation per unit of (informal) capital used in production - the average tax rate increases from 0.03 ($\zeta^* = 1$) to 0.05 ($\zeta^* = 1.5$).

In a less tolerant society ($\zeta = 1.5$), informal entrepreneurs reduce the amount of labor input more than the use of capital input in production. While the former falls from 37.45 in the bench-



Figure 2: Distribution of Agents Regarding Talent, Wealth and Income

Notes: Simulations are based on a random draw of 20,000 individuals from the distributions of talent and wealth described previously. The intervals of each of the vectors in the equilibrium are $\Pi \in [0.19846, 0.84271]$, $b \in [0.023469, 0.1036]$ and $x \in [0, 2]$.

mark to 35.82, the latter falls from 15.40 to 14.80. In this case, production in the formal (informal) sector increases (decreases) by about two (three) percent. Interestingly, as the society becomes less tolerant with informality, the observed production increase in the formal sector occurs mostly through the intensive margin channel - formal entrepreneurs benefit from a lower equilibrium wage that more than compensates for the increase in the interest rate. On the other hand, the decrease in the informal sector production is due to fewer agents working less hours (both extensive and intensive margins). Thus, the increase in the government's tax revenue is mainly due to the higher production in the formal sector, which more then compensates the drop in the revenue collected from the informal sector.

Regarding the distribution of occupational choices, we observed that changes in the tolerance parameter lead agents to move across occupations - informal entrepreneurs become workers rather than across the formal-informal sector dimension. For instance, the less tolerant a society is the larger the drop in the share of informal firms - e.g., from 14.33 in the benchmark case $(\zeta = 1.0)$ to 13.19 ($\zeta = 2.0$); Table III. This drop is accommodated by a reallocation of agents from entrepreneurship to paid work and it is reflected in a higher share of workers in the economy - from 76.72 ($\zeta = 1.0$) to 77.89 ($\zeta = 2.0$). Changes in the share of formal entrepreneurs are very small, which can be attributed, for instance, to technological barriers as well as financial constraints.

The last two columns of Table III present a more meaningful counterfactual exercise. If we set $\zeta = 4.60$ ($\zeta = 7.72$) we can match the size of the informal sector in countries like Italy (United States), which according to Medina and Schneider (2018) is about 23.61% (7.75%) of the GDP. Overall, similar qualitative effects are observed. In particular, we notice the impact of a society's lower tolerance of informal activities (higher ζ) in the composition of the economy production sector. Although aggregate production declines, mainly due to the sharp drop in the informal sector production, the average entrepreneurial ability of those managing firms in the economy increases. In other words, we observe more high ability agents in the entrepreneur occupations (formal, informal) in less tolerant societies. Finally, compared to our benchmark value ($\zeta = 1$), all these results highlights that the size of the informal sector can be associated with how a society perceives and tolerates informal activities. As $\zeta \to \infty$, i.e. an intolerant society, the size of the informal sector converges to zero.

Labor Income and Formal Output Taxation. Compared to our benchmark case, the immediate effect of a lower tax on labor income τ_w is an increase in the worker (net and gross) wage income, making this occupation more attractive in both sectors. So from the agent's point of view, a reduction of taxes on wages has an important effect regarding an occupational choice rearrangement: we observe that informal entrepreneurs change their occupation to become workers, as this now represents a higher payoff. Consider, for instance, a lower labor income tax relative to our (Brazil) benchmark. For instance, let $\tau_w = 0.22$, similar to the wage taxation in countries like Israel and Switzerland (OECD, 2020b). In this case, the share of workers in the population is 77.94 (up from 76.72 in the benchmark), the informal entrepreneurs share is 13.09 (down from 14.33) and the share of formal entrepreneurs in the economy remains the same. Entrepreneurs move from the informal sector to wage employment, a sector that now has slightly higher payoff on average.

The fact that some informal entrepreneurs change occupation and become workers leads to (i) a lower demand for capital in the informal sector and (ii) a higher supply of this factor by workers. As a result, the interest rate falls by approximately 1.4%. A lower interest rate benefits entrepreneurs in the formal sector - a sector that is relatively more productive and less labor intensive. In other words, the marginal benefit of a lower labor income taxation is higher in the formal sector, which leads formal entrepreneurs to hire more capital and to produce more output. As a matter of fact, the observed increase in the formal output is due to changes in the intensive margin only. That is, formal entrepreneurs can scale their production up as capital becomes cheaper. Although the tax base increases and the size of the informal sector falls, the net government revenue change is negative and tax collection decreases.

Table V presents the effects on key variables of changes in the taxation of formal output

$ \begin{array}{c cccc} \zeta = 0 & \zeta = 0.5 & \zeta^* = 1 & \zeta = 1.5 & \zeta = 2 & \zeta = 4.6 & \zeta = 7.5 \\ \hline Occupational Choice (\%) \\ \hline Workers & 75.84 & 76.28 & 76.72 & 77.22 & 77.89 & 80.69 & 86.22 \\ \hline Entrepreneurs & & & & \\ \hline Formal & 8.97 & 8.91 & 8.95 & 8.85 & 8.92 & 9.55 & 10.51 \\ \hline Informal & 15.20 & 14.82 & 14.33 & 13.93 & 13.19 & 9.77 & 3.26 \\ \hline \end{array} $	7			L			
$\begin{array}{c c c c c c c c c c c c c c c c c c c $	ς	$= 0 \zeta = 0.5$	$\zeta^* = 1$	$\zeta = 1.5$	$\zeta = 2$	$\zeta = 4.6$	$\zeta = 7.7$
Workers 75.84 76.28 76.72 77.22 77.89 80.69 86.22 Entrepreneurs Formal 8.97 8.91 8.95 8.85 8.92 9.55 10.51 Informal 15.20 14.82 14.33 13.93 13.19 9.77 3.26	ional Choice (%)						
Entrepreneurs 8.97 8.91 8.95 8.85 8.92 9.55 10.51 Informal 15.20 14.82 14.33 13.93 13.19 9.77 3.26	75	5.84 76.28	76.72	77.22	77.89	80.69	86.22
Formal 8.97 8.91 8.95 8.85 8.92 9.55 10.51 Informal 15.20 14.82 14.33 13.93 13.19 9.77 3.26	neurs						
Informal 15.20 14.82 14.33 13.93 13.19 9.77 3.26	rmal 8	8.97 8.91	8.95	8.85	8.92	9.55	10.51
	ormal 15	5.20 14.82	14.33	13.93	13.19	9.77	3.26
Formal Sector	<u>bector</u>						
Output (y_F) 60.49 62.05 63.78 64.95 66.84 76.39 92.25	(y_F) 60	0.49 62.05	63.78	64.95	66.84	76.39	92.25
Capital input (k_F) 82.89 83.70 84.60 85.20 86.29 90.94 97.40	nput (k_F) 82	2.89 83.70	84.60	85.20	86.29	90.94	97.40
Labor input (l_F) 58.40 60.42 62.55 64.18 66.33 76.98 92.88	put (l_F) 58	8.40 60.42	62.55	64.18	66.33	76.98	92.88
Informal Sector	Sector						
Output (y_I) 39.5137.9536.2235.0533.1623.617.75	(y_I) 39	9.51 37.95	36.22	35.05	33.16	23.61	7.75
Capital input (k_I) 17.11 16.30 15.40 14.80 13.19 9.06 2.60	nput (k_I) 17	7.11 16.30	15.40	14.80	13.19	9.06	2.60
Labor input (l_I) 41.60 39.58 37.45 35.82 33.67 23.02 7.12	put (l_I) 41	1.60 39.58	37.45	35.82	33.67	23.02	7.12
Tax rate (average) 0.0 0.02 0.03 0.05 0.06 0.11 0.16	(average)	0.0 0.02	0.03	0.05	0.06	0.11	0.16
The collection 1	ation1						
$\frac{12X \text{ Collection}}{W_{\text{excloser}}} = 15.91 15.42 15.40 15.10 15.24 14.92 15.01$	<u>2CUIOII</u> 11	F01 1F49	15 40	15 10	15.94	14.09	15 01
Workers 15.81 15.43 15.40 15.19 15.34 14.83 15.01	10	5.81 15.45	15.40	15.19	15.54	14.85	15.01
Entrepreneurs	neurs	0.57 01.10	01 00	00.00	00 79	05 07	01.07
Formal 20.57 21.10 21.68 22.08 22.73 25.97 31.37 L 0.0 0.00 1.14 1.71 1.00 0.04 1.07	mal 20	0.57 21.10	21.68	22.08	22.73	25.97	31.37
Informal $0.0 0.62 1.14 1.71 1.96 2.64 1.26$	ormal	0.0 0.62	1.14	1.71	1.96	2.64	1.26
% Variation $(\triangle)^2$	tion $(\triangle)^2$						
$\frac{1}{\text{Output } (u_F + u_I)} = -1.31 0.70 0.0 0.08 -1.04 -3.02 -5.79$	$\overline{(y_F + y_I)}$ -1	1.31 0.70	0.0	0.08	-1.04	-3.02	-5.79
Formal Output (y_F) -6.39 -2.03 0.0 1.93 3.71 16.17 36.28	$\operatorname{Dutput}\left(u_{F}\right) \qquad -6$	6.39 -2.03	0.0	1.93	3.71	16.17	36.28
Informal Output (y_I) 7.64 5.49 0.0 -3.17 -9.41 -36.79 -79.85	Output (y_I)	7.64 5.49	0.0	-3.17	-9.41	-36.79	-79.85
Consumption 0.36 0.93 0.0 -0.78 -1.43 -4.76 -5.32	otion (31)	0.36 0.93	0.0	-0.78	-1.43	-4.76	-5.33
Tax Collection -3.21 -0.82 0.0 0.59 1.57 6.69 17.81	ection -:	3.21 -0.82	0.0	0.59	1.57	6.69	17.81
Wage income ³ $2.44 \ 1.46 \ 0.0 \ -1.95 \ -2.93 \ -11.23 \ -18.31$	come ³	2.44 1.46	0.0	-1.95	-2.93	-11.23	-18.31
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	rate -6	6.64 -3.81	0.0	5.08	$\frac{1.00}{7.91}$	28.12	43.06

Table III: Tolerance of Informality (ζ)

Notes: *benchmark model; (1) Total tax revenue from each occupation (% of total GDP); (2) % variation relative to the benchmark case; (3) Gross wage income.

 τ_F . As expected, decreases in the formal output tax rate τ_F increase the profitability of formal sector entrepreneurs. A lower tax on output leads to two distinct changes. First, it becomes more attractive for informal entrepreneurs to become formal entrepreneurs, moving to a less labor-intensive production technology. And, second, it leads entrepreneurs already operating in the formal sector to expand their production levels, i.e., to hire more workers and to use more capital. Let $\tau_F = 0.28$ - a taxation similar to the one observed in countries like Sweden, Norway or Italy.¹³

¹³Taxation trends in the European Union. See also Global Revenue Statistics Database, OECD.Stat and OECD (2020a).

	$\tau = 0.22$	$\tau = 0.25$	$\tau^* - 0.275$	$\tau = 0.30$	$\tau = 0.33$
Occupational Choice $(\%)$	$T_w = 0.22$	$T_w = 0.20$	$T_w = 0.210$	$T_{W} = 0.50$	$T_w = 0.00$
Occupational Choice (70)					
Workers	77.94	77.24	76.72	76.39	75.50
Entrepreneurs					
Formal	8.96	8.96	8.96	8.82	8.81
Informal	13.09	13.80	14.33	14.80	15.69
Production					
Formal (y_F)	65.71	64.69	63.78	62.75	61.37
Informal (y_I)	34.29	35.51	36.22	37.25	38.63
Tax collection ¹					
Workers	12.40	13.93	15.40	16.93	18.38
Entrepreneurs					
Formal	22.34	21.93	21.68	21.34	20.87
Informal	1.14	1.15	1.14	1.15	1.16
% Variation $(\triangle)^2$					
Output $(y_F + y_I)$	1.49	1.24	0.0	-1.18	-1.51
Formal Output (y_F)	4.57	2.38	0.0	-2.77	-5.22
Informal Output (y_I)	-3.93	-0.76	0.0	1.61	5.03
Consumption	5.69	2.66	0.0	-2.53	-5.54
Tax Collection	-4.94	-2.13	0.0	1.96	4.22
Wage $income^3$	0.49	0.00	0.0	0.0	-0.49
Interest rate	-1.42	-0.29	0.0	0.68	1.71

Table IV: Labor Income Tax (τ_w)

Notes: *benchmark model; (1) Total tax revenue from each occupation (% of total GDP); (2) % variation relative to the benchmark case; (3) Gross wage income. The labor income taxes were chosen for counterfactual purposes to resemble tax rates in the following countries: Israel ($\tau_w = 0.22$), Korea ($\tau_w = 0.25$), United Kingdom ($\tau_w = 0.30$) and Iceland ($\tau_w = 0.33$); Tax wedge of a single worker without children earning a nation's average wage (OECD, 2020b).

Relative to our benchmark value $\tau_F = 0.34$, a decrease in the formal output taxation of six percentage points ($\tau_F = 0.28$) causes a substantial decrease in the share of informal entrepreneurs in the labor force. It drops from 14.33 in the benchmark case ($\tau_F = 0.34$) to 10.15 ($\tau_F = 0.28$). We also observe an increase in the share of formal entrepreneurs in the economy (from 8.96 to 9.37, respectively).

Comparing the effect of changes in these two tax instruments (τ_w, τ_F) on the government tax revenue, we observe that while reductions of τ_w lead to a drop in the government tax revenues (Table IV), tax revenue actually increases by reducing taxation of the formal output (τ_F) (Table V). This result might suggest that, regarding the taxation of output produced in the formal sector, the Brazilian economy is on the "wrong side" of the Laffer curve.¹⁴

¹⁴The "wrong side" of the Laffer Curve denotes a situation in which the tax rate is greater than the one that

	$\pi_{-} = 0.28$	$\pi_{-} = 0.21$	$\pi^* = 0.24$	$\tau_{-} = 0.27$	$\pi_{-} = 0.40$
	$T_F = 0.28$	$T_F = 0.31$	$T_F = 0.34$	$T_F = 0.37$	$T_F = 0.40$
Occupational Choice $(\%)$					
Workers	80.47	78.47	76.72	74.88	73.22
Entrepreneurs					
Formal	9.37	9.03	8.96	8.64	8.52
Informal	10.15	12.51	14.33	16.48	18.27
Production					
1100000000000000000000000000000000000	74.87	68 67	63 78	57 60	59 13
I = (1 + 1)	05 19	00.07	20.10	40.01	17.07
Informal (y_I)	25.13	31.33	30.22	42.31	47.87
<u>Tax collection¹</u>					
Workers	15.40	15.31	15.40	15.34	15.67
Entrepreneurs					
Formal	20.96	21.29	21.68	21.35	20.85
Informal	0.85	1.01	1.14	1.28	1.33
% Variation $(\triangle)^2$					
$\overline{\text{Output}(y_F+y_I)}$	8.55	3.89	0.0	-3.90	-10.30
Formal Output (y_F)	27.43	11.87	0.0	-13.07	-26.68
Informal Output (y_I)	-24.70	-10.15	0.0	12.24	18.54
Consumption	9.41	3.96	0.0	-4.54	-10.23
Tax Collection	6.41	2.53	0.0	-4.94	-11.67
Wage $income^3$	3.42	0.98	0.0	-1.95	-4.39
Interest rate	26.85	15.09	0.0	-13.08	-25.97

Table V: Formal Output Tax (τ_F)

Notes: *benchmark model; (1) Total tax revenue from each occupation (% of total GDP); (2) % variation relative to the benchmark case; (3) Gross wage income. The formal output tax rates (τ_F) were chosen for counterfactual purposes to resemble the tax revenue as share of GDP in the following countries: Sweden ($\tau_F = 0.28$), Italy ($\tau_F = 0.31$), United States ($\tau_F = 0.39$), and Japan ($\tau_F = 0.42$); (Source: Taxation trends in the European Union.)

Accessibility to the Financial Markets. A higher accessibility to the financial markets by formal entrepreneurs has three main effects when compared to our benchmark case $\lambda_F = 10$ (Table VI). First, formal sector entrepreneurs have more access to credit in order to finance their production. This implies a drop in the informal sector production, which is more than compensated by the formal output production increase (scale production up). The overall effect of more access to additional funds leads to more production in the more efficient sector and, consequently, more output and consumption. Second, the (gross) equilibrium wage (w) increases, making the worker occupation more attractive for some informal entrepreneurs. Through this channel the size of the informal sector falls in both the output and the employment dimensions. Higher wages and

would maximize total tax revenue (Trabandt and Uhlig (2011)). A reduction in the current tax rate would, hence, increase the tax revenue.

higher formal output combined increase the government tax base. Hence, the government's labor income and output tax revenues increase - a result in line with Lopez-Martin (2019).¹⁵ Finally, due to a greater availability of capital, a lower interest rate stimulates the expansion of production by the formal sector, which is more capital intensive. Overall, changes in the financial markets accessibility lead to changes in the occupational choice shares in the economy.

	$\lambda_{\rm T} - 8$	$\lambda_{\rm P} = 0$	λ^* – 10	$\lambda_{\rm E} - 11$	$\lambda_{\rm T} - 12$
(Credit % GDP)	$\chi_F = 0$ (26.31)	$\chi_F = 3$ (30.52)	(34.00)	(37.39)	(42.58)
	(20:01)	(00.02)	(01:00)	(01.00)	(12.00)
Occupational Choice (%)					
Workers	75.10	76.08	76.72	77.59	78.45
Entrepreneurs					
Formal	8.12	8.84	8.96	9.09	9.48
Informal	15.78	15.07	14.33	13.32	12.07
Production					
Formal (y_F)	58.34	60.65	63.78	67.10	71.34
Informal (y_I)	41.66	39.35	36.22	32.90	28.66
-					
<u>Tax collection¹</u>					
Workers	15.45	15.53	15.40	15.42	15.32
Entrepreneurs					
Formal	19.84	20.62	21.68	22.81	24.26
Informal	1.30	1.24	1.14	1.06	0.93
% Variation $(\Delta)^2$					
Output $(y_F + y_I)$	-4.54	-2.60	0.0	2.56	5.81
Formal Output (y_F)	-12.68	-7.38	0.0	7.91	18.36
Informal Output (y_I)	9.79	5.82	0.0	-6.86	-16.28
Consumption	-1.50	-0.53	0.0	0.48	0.82
Tax Collection	-9.18	-5.06	0.0	5.71	12.92
Wage $income^3$	-2.20	-0.98	0.0	1.46	2.93
Interest rate	2.73	0.68	0.0	-0.78	-2.34

Table VI: Entrepreneur's Accessibility to Financial Markets (λ_F)

Notes: *benchmark model; (1) Total tax revenue from each occupation (% of total GDP); (2) % variation relative to the benchmark case; (3) Gross wage income. Using the World Bank Global Financial Inclusion Database and by means of counterfactual, higher (lower) accessibility of formal entrepreneurs to the financial markets can be associated to countries, such as the Chile, Poland, and France (Argentina, Peru, Paraguay, and other developing countries).

¹⁵The share of workers increases when the financial markets is more accessible can be attributed to a increase in the equilibrium wage, which makes the worker occupation relatively more attractive. While a higher λ_F makes it easier for formal entrepreneurs to access additional funds to finance their production, it also increases, in equilibrium, the returns for those that choose to be workers. Intuitively, higher access to credit in the formal sector increases the demand for labor and the equilibrium wage level.

3.3 Stochastic taxation of informal output

We have assumed so far in our theoretical model (Section 2) and in our numerical exercises (Sections 3.1, 3.2) that the taxation of informal production is deterministic. In the quantitative exercises presented and discussed in Subsection 3.2, informal entrepreneurs are subject to the same tolerance parameter ζ , and the informal output tax is

$$\tau_I = 1 - e^{-k_I \zeta}.$$

We now extend the model to consider a stochastic taxation of informal activities. That is, being caught by the tax authority managing an informal production technology is a stochastic event. All informal entrepreneurs are inspected and they are forced to pay a tax that depends on the size of the firm (as before) and on tax auditors (heterogeneous) tolerance of informal activities. In this environment, informal entrepreneurs face the same probability of being caught by either a more (low ζ) or a less tolerant (high ζ) tax auditor. In other words, while some tax auditors are more tolerant and consequently will impose a lower tax on informal entrepreneurs, others are stricter (less tolerant) and will impose higher tax rates on informal entrepreneurs. Therefore, the payoff of an informal entrepreneur now depends not only on her entrepreneurial ability and wealth but also on a probability of being inspected and on the tolerance level of a tax auditor.¹⁶

Assume that the society's tolerance of informality (ζ) interpreted now as the tax auditor tolerance of informality follows a distribution $\Gamma(\zeta)$ with support in the interval $[\zeta_L, \zeta_H]$, where ζ_L and ζ_H represent the lower- and upper-level of tolerance, i.e., tax auditors that are more and less tolerant of informal activities. Hence, an informal entrepreneur is subject to the following (expected) informal tax rate

$$\tau_I = \int_{\zeta_L}^{\zeta_H} (1 - e^{-k_I \zeta}) d\Gamma(\zeta), \qquad (27)$$

and her (stationary) expected profit is given by $\int_{\zeta_L}^{\zeta_H} \pi_{e,I}(b,x;w,r,\zeta) d\Gamma(\zeta)$.

We conduct two numerical experiments in this section. In the first experiment, we assume that ζ is drawn from a uniform distribution $\mathcal{U}(0, 2)$ and, in the second experiment, it is drawn from an alternative uniform distribution $\mathcal{U}(0.5, 2.5)$. To gain some intuition, these values are equivalent to an informal output tax that ranges between 0-5% in the first case and 1.5%-7%, in the second one. Thus, while in the first numerical experiment informal entrepreneurs might not be taxed at all (i.e., an informal tax equals to zero), in the $\zeta \sim \mathcal{U}(0.5, 2.5)$ case, tax auditors are (on average) less tolerant of informal activities. We assume that the expected value of ζ is such that her expected informal tax rate is equal to the one she faces in the deterministic case (Section 2). In both cases, informal entrepreneurs face uncertainty regarding the tax auditor's tolerance of informal activities,

¹⁶We acknowledge that there are other way to model a stochastic taxation or punishment of informal activities. With the proposed extension we change the model only parsimoniously while keeping its main features. The level of tolerance of tax auditors can be, for instance, associated with the size of the informal firm, the kind of output produced, levels of corruption and side payments that are not modelled directly in our work.

i.e., whether a more or less tolerant tax auditor will inspect their businesses. We also assume that the entrepreneurs' (b, x) profiles and the auditors' tolerance levels (ζ) are not correlated.

Table VII presents our results. To allow for comparisons, the results for our benchmark deterministic taxation of informal output (Table III, Subsection 3.2) are also presented in Table VII. Overall, uncertainty regarding informal output taxation reduces informality. In particular, when the society (tax auditors) is less tolerant of informal activities the share of informal entrepreneurs and informal production are smaller relative to the benchmark case $\zeta^* = 1$. On the other hand, formal output is higher and government tax revenue increases.

The sharpest contrast between these two exercises is in the equilibrium interest rate. In the case tax auditors are (on average) as tolerant of informal activities as in the deterministic case, i.e., $\zeta \sim \mathcal{U}(0, 2)$, but informal entrepreneurs face an stochastic taxation (punishment), the equilibrium interest rate is smaller. As entrepreneurs leave the informal sector to become workers, this occupational reallocation reduces the demand for capital and pushes the economy interest rate to a lower level (-1.37%). For a higher level of uncertainty $\zeta \sim \mathcal{U}(0.5, 2.5)$, the reduction of total output $(y_F + y_I)$, as well as the share of entrepreneurs in the labor force, is more pronounced. And, while we observe no variation in the equilibrium wage rate in the first case, i.e., $\zeta \sim \mathcal{U}(0, 2)$, it falls substantially in the second case $\zeta \sim \mathcal{U}(0.5, 2.5)$. The fall in production (both in the intensive and extensive margins), along to a lower equilibrium wage, leads to a higher equilibrium interest rate when informal entrepreneurs face a higher level of uncertainty regarding the taxation of their output and they manage firms in a society that is less tolerant to their activities.

4 Conclusions

In this paper we show that the endogenous taxation of informal output has important implications for the allocation and production of output in both the formal and the informal sectors of an economy, as well as for the agents' occupational choices (entrepreneurs vs. workers). We develop a framework where an entrepreneur that manages an informal firm is subjected to a tax rate that is determined by the combination of her capital choice and the society's tolerance of informality. The latter is the main novelty of the paper. In our theoretical model and quantitative exercises, we study the joint effects of how a society tolerates informal production (social norms) and how informal entrepreneurs themselves perceive the punishment imposed by the government, which is reflected on their (endogenous) choice of capital input. The combination of these two features affects the informal entrepreneur's maximization problem and, hence, the general equilibrium effects of policy changes.

Our model is consistent with many empirical findings regarding the informal sector in Brazil, a developing economy with a large informal sector. With a calibrated version of our model, we show that as the society's tolerance of informality decreases, the informal sector employs less capital and labor inputs and produces less output - informality decreases. Because the taxation

	$\zeta^* = 1$	$\zeta \sim \mathcal{U}(0,2)$	$\zeta \sim \mathcal{U}(0.5, 2.5)$
Occupational Choice (%)	J	3 \//	3 () - /
Workers	76.72	76.91	77.68
Entrepreneurs			
Formal	8.96	8.93	9.00
Informal	14.33	14.16	13.32
Formal Sector			
$\frac{\text{Format Sector}}{\text{Production }(u_{\text{P}})}$	63 78	63 80	65.83
Capital $(k_{\rm p})$	84.60	84 78	85.83
Labor (l_{r})	62 55	62.70 62.57	64.01
Labor (l_F)	02.00	02.57	04.91
Informal Sector			
$\overline{\text{Production } (y_I)}$	36.22	36.11	34.17
Capital (k_I)	15.40	15.22	14.17
Labor (l_I)	37.45	37.43	35.09
Average tax rate	0.03	0.03	0.04
Tax collection ¹			
Workers	$15 \ 40$	15 48	15 52
Entrepreneurs	10.10	10.10	10.02
Formal	21.68	21.72	22.38
Informal	1.14	1.03	1.47
07 Variation $(\wedge)^2$			
$\frac{70}{\text{Output}} \left(\frac{4}{2} + \frac{4}{2} \right)$	0.0	0.27	0.07
Formal Output $(y_F + y_I)$	0.0	-0.27	-0.97
Formal Output (y_F)	0.0	-0.09	2.21 6 50
$\begin{array}{c} \text{Informal Output} \ (y_I) \\ \text{Output} \ (y_I) \end{array}$	0.0	-0.38	-0.59
Consumption	0.0	0.11	-0.04
Tax Collection	0.0	0.05	1.20
vvage income ^o	0.0	0.00	-1.40
Interest rate	0.0	-1.37	3.12

Table VII: Stochastic Tolerance of Informality

Notes: *benchmark model; (1) Total tax revenue from each occupation (% of total GDP); (2) % variation relative to the benchmark case; (3) Gross wage income.

of informal activities is endogenously determined by how much capital informal entrepreneurs use, a less tolerant society imposes a higher taxation per unit of (informal) capital used. We also observe that changes in the society's tolerance of informality lead agents to shift between informal entrepreneurship and salaried work rather than the two entrepreneurial choices (formal and informal). Overall, our results show that informality is substantially lower (while output, consumption and tax collection are higher) in economies that are less tolerant of informal activities, formal entrepreneurs have more access to financial markets and taxation of output and labor is lower. We also extend the model to consider a stochastic taxation of informal activities and we show that uncertainty regarding informal output taxation reduces informality.

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